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Renormalizable mean field calculation in QED with fermion background

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Abstract

We study quantum electrodynamics coupled to the matter field on a singular background, which we call defect. For defect on an infinite plane we calculated the mean electromagnetic field. Quantum corrections determining the field near the plane are calculated in the leading order of perturbation theory. We analyse the normalization conditions for the parameters of the defect and calculate the photoelectric function of the charged particle from the defect.

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1. Introduction

Theory of Casimir effect based on the assumption that it is a macroscopic phenomenon generated by vacuum fluctuations of quantum electrodynamic (QED) fields is in good agreement with experimental data [1–3]. It is natural to expect that the Casimir effect is not a unique macro-manifestation of quantum fields' fluctuations, and in many other problems description of macro-system behaviour obtained by classical electrodynamics (ED) needs essential quantum corrections. The calculation of them is a theoretical task being of practical importance for development of nanotechnologies and design of microdevices [4].

A typical ED statement of a problem is to find electric and magnetic fields for the given boundary conditions, charge and current distributions. In this paper, we consider the QED version of the simplest problem of such a kind. We study gauge invariant, local, renormalizable model of a simple defect on the plane. It is suggested for calculation of quantum corrections for the fields of a plane in classical electrodynamics. It is essential that for renormalizability of the model the direct interactions of the boundary both with the photon and with the Dirac fields are necessary. We calculate the leading order approximations for mean strengths of electric and magnetic fields expressed by usual relations in terms of the components of the electromagnetic field tensor. Results look like classical ones at large distance from the plane. For small distances $r \to 0$, the strength of fields appears to be singular as $C_1/r^2 + C_2/r$, where C_1, C_2 are constants. It is essentially a non-classical effect generated by the interaction of the Dirac fields with the defect on the plane.

2. Statement of the problem

The full action of the model is a sum of QED bulk action and the most general surface action of photon and fermion defects consistent with first principles of QED: gauge invariance, locality, renormalizability: $S(A,\Phi) = S_{\rm QED} + S_\Phi^{(1)} + S_\Phi^{(2)} + S_\Phi^{(3)}$, where

$$S_\Phi^{(1)} = \frac{a}{2}\int \varepsilon^{\lambda\mu\nu\rho}\partial_\lambda\Phi(x)A_\mu(x)F_{\nu\rho}(x)\delta(\Phi(x))\,{\rm d}x,$$

$$S_\Phi^{(2)} = \int \bar\Psi(x)\hat{Q}\Psi(x)\delta(\Phi(x))\,{\rm d}x, \qquad \hat{Q} = Q_{[\mu]}\Gamma_{[\mu]},$$

$$S_\Phi^{(3)} = \int \big(lA(x) + l'\partial_n^2A(x) + \tilde{l}\partial_nA(x)\big)\delta(\Phi(x))\,{\rm d}x, \qquad (nl) = (nl') = (n\tilde{l}) = 0,$$

$\Phi(x) = 0$ is the equation of the surface and $\Gamma_{[\mu]}$ is a full set of 16 Dirac matrices: $\Gamma_{[\mu]} = (I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu})$, ∂_n is the derivative normal to the surface and n is the normal unit vector. The photon defect $S_\Phi^{(1)}$ was studied in [5]. The fermion defect $S_\Phi^{(2)}$ was partially considered in [6]. The $S_\Phi^{(3)}$ action describes the field of external charges and currents. The effect due to photon defect exhibits itself in a long-range macroscopical fluctuation relevant on microscale. The effects of fermion defect are exponentially suppressed on distances larger than the electron Compton wavelength.

In this paper, we restrict ourselves only to the fermion defect concentrated on an infinite plane $x_3 = 0$ invariant with respect to coordinate reflections. Thus, we exclude from consideration all parity violating terms of the action (containing γ_5, $\tilde{l}$). The tensor $\sigma_{\mu\nu}$ in fermion defect describes magnet moments density of the surface. We assume it to be zero which can be done without losing renormalizability. Then the model is specified by the following action functional:

$$S(\bar\psi, \psi, A) = S_{\rm QED}(\bar\psi, \psi, A) + S_{\rm def}(\bar\psi, \psi, A; \lambda, q, \xi), \qquad (1)$$

where $S_{\rm QED}(\bar\psi, \psi, A)$ is the usual QED action

$$S_{\rm QED}(\bar\psi, \psi, A) = \int \bar\psi(x)({\rm i}\hat\partial - e\hat{A}(x) - m)\psi(x)\,{\rm d}x - \frac{1}{4}\int F_{\mu\nu}(x)F^{\mu\nu}(x)\,{\rm d}x$$

and $S_{\rm def}(\bar\psi, \psi, A; \lambda, q, l) = S_{\lambda q}(\bar\psi, \psi) + S_l(A)$ with

$$S_{\lambda q}(\bar\psi, \psi) \equiv \int \bar\psi(\vec{x}, 0)(\lambda + \hat{q})\psi(\vec{x}, 0)\,{\rm d}\vec{x} \qquad S_l(A) \equiv \int lA(\vec{x}, 0)\,{\rm d}\vec{x} + \int l'\partial_3^2A(\vec{x}, 0)\,{\rm d}\vec{x}.$$

Here $\bar\psi$, ψ are the Dirac spinor fields, A is the electromagnetic vector potential, q, l, l' are fixed 4-vectors, $\hat{q} = q_\mu\gamma^\mu$ (γ^μ are the Dirac gamma-matrices), and we used the short-hand notation for the 4-vector $x : x = (x_0, x_1, x_2, x_3) = (\vec{x}, x_3)$. The notation of this kind will also be used later. For gauge invariance of the defect action, it is necessary to set $l_3 = l'_3 = 0$.[3] The model remains renormalizable for $l' = 0$ too, because as we will see below, there are no divergences which need for their cancellation the l'-term in S_l.

The physical meaning of the vector $l = (\vec{l}, 0)$ is very simple. It defines the classical 4-current on the defect plane. By neglecting in our model interaction of the photon and Dirac fields, the mean electromagnetic field coincides with the solution of Maxwell equations with the current (supported on the plane) defined by S_l. Vector q and scalar λ describe the interaction of the current and density of the Dirac field with material defect. Namely, the zero component of vector $\vec{q}$ defines a surface charge density and spacelike components of vector $\vec{q}$

[3] We consider the gauge transformations $A_\mu \to A_\mu + \partial_\mu\phi$ not changing the asymptotic of the field $A_\mu(x)$ for large x. It follows from this assumption that $\lim_{x_i\to\pm\infty}\phi(x) = \phi_0$, where ϕ_0 is a constant being the same for all $i = 0, 1, 2, 3$. Therefore $S_l(A)$ is gauge invariant.

parallel to the defect plane describe the surface current. The scalar λ defect can be interpreted as a surface mass term. The interaction of vacuum fluctuations of the Dirac field with the background generates quantum corrections to usual classical effects.

We calculate the leading approximation for the electromagnetic field generated by the defect. Only the l-term in S_l appears to be necessary for cancellation of ultraviolet divergences in our results. We set $l' = 0$ and do not consider a trivial contribution to the first-order effects from the l'-term in S_l. We choose the vector $\vec{l}$ proportional to $\vec{q}, \vec{l} = \vec{q}\xi$, and show that this 'minimal' form of S_l with only one extra to $S_{\lambda q}$ parameter ξ provides the cancellation of divergences by renormalization of ξ.

In this paper we study the mean tensor $\mathcal{F}_{\mu\nu}$ of the electromagnetic field:

$$\mathcal{F}_{\mu\nu} = C \int F_{\mu\nu}\, \mathrm{e}^{\mathrm{i}S(\bar\psi,\psi,A)} DAD\bar\psi D\psi, \qquad C^{-1} = \int \mathrm{e}^{\mathrm{i}S(\bar\psi,\psi,A)} DAD\bar\psi D\psi. \tag{2}$$

The tensor $F_{\mu\nu}$ is gauge invariant; therefore, $\mathcal{F}_{\mu\nu}$ is independent of the choice of the gauge. We provide calculations in Feynman's gauge using the formula

$$D_{\mu\nu}(x, y) = \frac{\delta_{\mu\nu}}{4\pi^2(x-y)^2}$$

for the photon propagator in configuration space. Integrating by parts in the functional integral (2) we obtain

$$\mathcal{F}_{\mu\nu}(x) = \frac{1}{2\pi^2} \int \mathrm{d}^4 y \frac{(x_\nu - y_\nu)J_\mu(y) - (x_\mu - y_\mu)J_\nu(y)}{(x-y)^4}, \tag{3}$$

where

$$J_\mu(y) = ej_\mu(y) - l_\mu\delta(y_3), \qquad j_\mu(y) = C\int \mathrm{e}^{\mathrm{i}S(\bar\psi,\psi,\vec{q})}\bar\psi(y)\gamma_\mu\psi(y)D\bar\psi D\psi. \tag{4}$$

In virtue of invariance of the action (1) with respect to translation of coordinates x_0, x_1, x_2 and reflection of x_3, $j_\mu(y)$ is an even function of coordinate y_3 only: $J(y) = J(y_3) = J(-y_3)$, and after integration over $\vec{y}$ in (3) we obtain the following result:

$$\begin{aligned}\mathcal{F}_{\mu\nu}(x) &= \frac{1}{2}\int_{-x_3}^{x_3} [\delta_{\nu 3}J_\mu(y_3) - \delta_{\mu 3}J_\nu(y_3)]\,\mathrm{d}y_3 \\ &= \mathrm{Sign}(x_3)\left\{e\int_0^{|x_3|}[\delta_{\nu 3}j_\mu(y_3) - \delta_{\mu 3}j_\nu(y_3)]\,\mathrm{d}y_3 - \xi(\delta_{\nu 3}q_\mu - \delta_{\mu 3}q_\nu)\right\},\end{aligned} \tag{5}$$

where $\mathrm{Sign}(x_3)$ is the signum function: $\mathrm{Sign}(x_3) = x_3/|x_3|$.

The vector j_μ in (4) is the current generated by vacuum fluctuations of Dirac fields. In virtue of Farri's theorem, it vanishes in the absence of defect. By usual methods of renormalization theory and its modification for the quantum field theory with singular background [7], one can prove that in the framework of renormalized perturbation theory $j_\mu(y_3)$ is presented by a sum of diagrams with all necessary subtractions. Therefore, it is finite (for the leading approximation it will be clear from the evident formula below), and in calculation of (5) there is only problem with non-integrable singularity of $j_\mu(y_3)$ at $y_3 = 0$. Therefore, the integral (5) needs a regularization. Since for $x_3 \neq 0$, the derivative of $\mathcal{F}_{\mu\nu}(x_3)$ with respect to x_3 is finite, we obtain the finite value of $\mathcal{F}_{\mu\nu}(x_3)$ subtracting from the integral on the left-hand side of (5) a constant dependent on the chosen regularization. This subtraction can be generated by the term S_l with the appropriate choice of the parameter ξ. Therefore, the l'-term in S_l is not necessary for renormalizability of the considered model.

3. Calculations and renormalization of $\mathcal{F}_{\mu\nu}$

The regularized form of the leading approximation of $\mathcal{F}_{\mu\nu}$ with the cut-off parameter Λ was obtained in [6]:

$$\mathcal{F}_{\mu\nu}(x) = e(q_\mu\delta_{\nu 3} - q_\nu\delta_{3\mu})\mathrm{Sign}(x_3)\left[F(x_3,\Lambda) - \frac{\xi}{2}\right],$$

where

$$F(x_3,\Lambda) \equiv \frac{1}{4\pi^2|\vec{q}|\alpha^2}\int_0^\Lambda \mathrm{d}p\frac{(1-\mathrm{e}^{-2E|x_3|})p}{E}\left[2p\alpha - (E(1-\alpha^2)+m\beta)\ln\frac{E+\alpha p+\beta m}{E-\alpha p+\beta m}\right],$$

$$\alpha = \frac{4|\vec{q}|}{4-q^2+\lambda^2},\qquad \beta = \frac{4\lambda}{4-q^2+\lambda^2},\qquad |\vec{q}| \equiv \sqrt{q_1^2+q_2^2-q_0^2},$$
$$q^2 \equiv q_0^2 - q_1^2 - q_2^2 - q_3^2.$$

The asymptotics of $F(\rho,\Lambda)$ for large Λ can be written as

$$F(\rho,\Lambda) = c_2\frac{\Lambda^2}{m^2} + c_1\frac{\Lambda}{m} + c_0 + f(\rho) + O\left(\frac{m^2}{\Lambda^2}\right),$$

where

$$f(\rho) \equiv -\frac{1}{4\pi^2|\vec{q}|\alpha^2}\int_0^\infty \mathrm{d}p\frac{\mathrm{e}^{-2E|\rho|}p}{E}\left[2p\alpha - (E(1-\alpha^2)+m\beta)\ln\frac{E+\alpha p+\beta m}{E-\alpha p+\beta m}\right],$$

$$c_2 = \frac{m^2}{8\pi^2|\vec{q}|\alpha^2}\left\{2\alpha + (\alpha^2-1)\ln\frac{1+\alpha}{1-\alpha}\right\},\qquad c_1 = \frac{\beta m^2}{4\pi^2|\vec{q}|\alpha^2}\left\{2\alpha - \ln\frac{1+\alpha}{1-\alpha}\right\}$$

and c_0 is an independent from Λ constant. The requirement that $\mathcal{F}_{\mu\nu}$ is finite for $\Lambda \to \infty$ means that ξ depends on Λ, and for large Λ its asymptotic has the form

$$\xi(\Lambda) = 2\left(c_2\frac{\Lambda^2}{m^2} + c_1\frac{\Lambda}{m}\right) + \chi + O\left(\frac{m^2}{\Lambda^2}\right).$$

Here, the parameter χ is the renormalized value of ξ.

Thus, if we denote $\vec{\mathcal{F}} = (\mathcal{F}_{03}, \mathcal{F}_{13}, \mathcal{F}_{23})$, then for $\Lambda \to \infty$ we obtain the following result:

$$\vec{\mathcal{F}}(x) = e\mathrm{Sign}(x_3)\vec{q}\left(c_0 + f(x_3) - \frac{\chi}{2}\right).$$

The asymptotics of the function $f(\rho)$ are of the form

$$f(\rho) \underset{\rho\to\infty}{=} -\frac{\alpha m^2\,\mathrm{e}^{-2m|\rho|}}{8|\vec{q}|(\pi m|\rho|)^{3/2}(1+\beta)}(1+O(1/\rho)),$$

$$f(\rho) \underset{\rho\to 0}{=} -\frac{c_2}{2m^2\rho^2} - \frac{c_1}{2m|\rho|} - c_0 + c_2 + O(\rho).$$

Hence,

$$\vec{\mathcal{F}}(x) \underset{x_3\to\infty}{=} e\mathrm{Sign}(x_3)\vec{q}\left(c_0 - \frac{\chi}{2} - \frac{\alpha m^2\,\mathrm{e}^{-2m|x_3|}}{8|\vec{q}|(\pi m|x_3|)^{3/2}(1+\beta)}(1+O(1/x_3))\right),$$

$$\vec{\mathcal{F}}(x) \underset{x_3\to 0}{=} -e\mathrm{Sign}(x_3)\vec{q}\left(\frac{c_2}{2m^2x_3^2} + \frac{c_1}{2m|x_3|} + \frac{\chi}{2} - c_2 + O(x_3)\right).$$

We see that at large distances from the plane, the field generated by the defect is of the same form as the field of the plane in classical ED, and at small distances x_3, it has non-classical behaviour $\vec{\mathcal{F}} \sim e\vec{q}c_2/2m^2x_3^2$.

Our model is invariant under translations and Lorenz transformations of $\vec{x}, \vec{q}$. Then we can classify the defect properties with the value of the invariant $|\vec{q}|$. Let us consider three cases: (1) $q = (\kappa, 0, 0, \tau) \equiv q^{(1)}$; (2) $q = (0, \kappa, 0, \tau) \equiv q^{(2)}$; (3) $q = (\kappa, \kappa, 0, \tau) \equiv q^{(3)}$. If $q = q^{(1)}$, the defect generates pure electric field: $H_1 = H_2 = H_3 = 0$, $E_1 = E_2 = 0$, $E_3 = -\mathcal{F}_{03}$. For $q = q^{(2)}$, the field is pure magnetic one: $E_1 = E_2 = E_3 = 0$, $H_1 = H_3 = 0$, $H_2 = -\mathcal{F}_{13}$. For $q = q^{(3)}$, there are both magnetic and electric fields: $E_1 = E_2 = H_1 = H_3 = 0$, $E_3 = H_2 = -\mathcal{F}_{13}$. We see that interaction of QED fields with the defect is described by four parameters: κ, $\tau \equiv q_3$, λ and χ.

4. Fields generated by simplest defects

The obtained results demonstrate the non-trivial dependence of the fields E and H on the parameters κ, λ, τ. In the main approximation, E and H are linear functions of χ and $E = H = 0$, if $\chi = 0, \kappa = 0$. Let us consider the simplest non-trivial case $\kappa \neq 0, \chi = \lambda = \tau = 0$. The asymptotics of E and H for large and small x_3 are as follows. If $q = q^{(1)}$, the defect generates the pure electric field E_3,

$$\begin{aligned} E_3 &\underset{x_3\to 0}{\approx} \frac{em^2}{8\pi^2\omega^2}[(1+\omega^2)\mathrm{arctg}(\omega) - \omega]\left(\frac{1}{m^2x_3^2} - 2\right). \\ E_3 &\underset{x_3\to\infty}{\approx} -\frac{em^2}{4\pi^2\omega^2}[\mathrm{arctg}(\omega) - \omega], \qquad \omega = \frac{4\kappa}{4-\kappa^2}. \end{aligned} \tag{6}$$

For $q = q^{(2)}$ the field is pure magnetic,

$$\begin{aligned} H_2 &\underset{x_3\to 0}{\approx} \frac{em^2}{16\pi^2\omega'^2}\left[(1+\omega'^2)\ln\frac{1+\omega'}{1-\omega'} - 2\omega'\right]\left(\frac{1}{m^2x_3^2} - 2\right). \\ H_2 &\underset{x_3\to\infty}{\approx} -\frac{em^2}{8\pi^2\omega^{2'}}\left[\ln\frac{1+\omega'}{1-\omega'} - 2\omega'\right], \qquad \omega' = \frac{4\kappa}{4+\kappa^2}. \end{aligned} \tag{7}$$

For $q = q^{(3)}$, $E_1 = E_2 = H_1 = H_3 = 0$ and the asymptotics of the fields E_3, H_2 are of the form

$$\begin{aligned} E_3 &\underset{x_3\to 0}{\approx} H_2 \underset{x_3\to 0}{\approx} \frac{e\kappa m^2}{12\pi^2}\left(\frac{1}{m^2x_3^2} - 2\right), \\ E_3 &\underset{x_3\to\infty}{\approx} H_2 \underset{x_3\to\infty}{\approx} \frac{e\kappa m^2}{12\pi^2}. \end{aligned} \tag{8}$$

The important feature of the fields generated by the considered defects for $q = q^{(1)}, q^{(2)}$ is that they are singular at $\kappa = \pm 2$. This means that these values of the parameter κ are the phase transition points, where in the case of $q = q^{(1)}$, the electrical field E_3 is changed suddenly and in the case of $q = q^{(2)}$, the field H_2 becomes infinite. This phenomenon seems to be not very surprising since it is similar to the known supercritical effects induced by perturbation of the Dirac field by the attractive δ potential which causes submerging of the ground state into the Dirac sea by the finite value of the coupling parameter [8–10].

The points $\kappa = \pm 2$ are stable with respect to the transformation $\kappa \to \kappa' = 4/\kappa$, for which $\omega(\kappa) \to \omega(\kappa') = -\omega(\kappa)$, $E_3(\kappa) \to E_3(\kappa') = -E_3(\kappa)$ and $\omega'(\kappa) \to \omega'(\kappa') = \omega'(\kappa)$, $H_2(\kappa) \to H_2(\kappa') = H_2(\kappa)$. For $q = q^{(1)}$ in each point of space the magnitude of field E_3, considered as a function of κ, is restricted: $|E(\kappa)| \leqslant |E(2)| < \infty$ for all values of κ.

For $\lambda = \chi = \tau = 0$ the short-distance asymptotic is of the form $E, H \sim \mathrm{const}[1/(m^2x^2) - 2]$. In this case, the relative correction of the next to leading term appears to be independent of the parameter κ describing specific properties of the defect on the plane.

Comparing the large distance asymptotics of (6) with the electric field of the charged plane in classical electrodynamics $E_{\rm cl} = \sigma/(2\varepsilon_0)$, we can identify the zeroth term of asymptotical expansion of E_3 with the classical field. Then κ is defined by the charge density per unit area σ as follows:

$$\sigma = -\frac{em^2\varepsilon_0}{2\pi^2\omega^2}[\mathrm{arctg}(\omega) - \omega], \qquad \omega = \frac{4\kappa}{4-\kappa^2}.$$

This is nonlinear equation on κ and it can be solved by standard numerical methods. We want to stress several features of this combination of the parameters. Firstly, the theory has four phases, i.e. for any given $\sigma < \sigma_{\rm critical}$, there are four different κ. Secondly, the fermion defect itself induces only finite surface charge density $\sigma < \sigma_{\rm critical}$ and photon defect is needed $(\zeta \neq 0)$ to induce arbitrary surface charge density σ.

According to the Biot and Savart law in classical ED the constant magnetic field is induced by an infinite plane with the constant current $\mathbf{J}$. Identifying the constant term of H_2 in (7) with the classical magnetic field, one can define κ via the current $\mathbf{J} = (0, 1, 0, 0)J$. In this case, the model has two phases and arbitrary values of J can be generated.

5. Conclusions

The suggested model describes an infinite plane with homogeneous charge and current distributions in the framework of QED. Specific properties of the physical system are characterized by an additional term $S_{\rm def}$ (action of defect) combined with the usual action of QED into the full action of the model. $S_{\rm def}$ was chosen on the basis of general principles of QED: locality, gauge invariance and renormalizability of the theory. The calculation of the leading order effects shows that the mean field induced by $S_{\rm def}$ has classical behaviour at a large distance from the plane. The corresponding asymptotic can be used as normalization conditions pinpointing interplay of parameters describing the fields of a plane in classical ED with those of the considered model. In this way, one can express four parameters of $S_{\rm def}$ in terms of the effective charge and current densities and constants characterizing macroscopic properties of the material of the plane.

At short distances the fields E, H are singular as functions of distance x from the defect: $E, H \sim \mathrm{const}/x^2$. Estimating the energy density with the usual classical formula $W = (E^2 + H^2)/2$, one obtains its behaviour for (6), (7) and (8) at short distances as $W(x) \sim c/x^4 + c'/x^2$, where c, c' are finite cut-off independent constants. These singularities representing physical peculiarities of the model could be predicted with dimensional analysis. It is similar to the one found for the scalar field under Dirichlet or Neumann boundary conditions on a single plate [11], akin local effects near surfaces can be observed in different geometries (see, for example, [12–14]). Our model predicts the dependence of c and c' on parameters of the material plane.

In our paper, we have restricted ourselves to the simple problem to calculate the mean electromagnetic field generated by perturbation of QED vacuum by an infinite plane film. It is important to note that a finite physical observable is extracted for a system of just one isolated plane in distinction to ordinary Casimir effect.

We hope that the proposed approach can provide a deeper insight into the nature of quantum phenomena of interaction of macroscopic bodies with QED fields. The direct experimental proof of the obtained results seems to be not easy because they concern the effects which are exponentially suppressed with the distance from the boundary (with the Compton wavelength of electron $\sim 10^{-10}$ cm as a characteristic scale). However, the electromagnetic fields formed by fermionic defect can play an essential role in many phenomena near boundary (output electron energy, spectrum of atoms, emission and scattering of electrons on the material boundary, etc). Therefore, we believe that there is a possibility of checking our results experimentally. We also expect that the suggested model will be useful for theoretical investigation in physics of two-dimensional materials.

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